২১/৮/২০১২৫ MATH 191

L1-/T-I/IPE Date: 24/08/2025

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-I B.Sc. Engineering Examinations 2024-2025

Sub: **MATH 191 (Differential and Integral Calculus)**

Full Marks: 280 Time: 3 Hours

The figures in the margin indicate full marks.

Symbols have their usual meanings.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Discuss the continuity and differentiability of the function **$(16\frac{2}{3})$**

$$f(x) = \begin{cases} 1+x & for\ x \le 0 \\ x & for\ 0 < x < 1 \\ 2-x & for\ 1 \le x \le 2 \\ 2x-x^2 & for\ x > 2 \end{cases}$$

at $x = 0$, $x = 1$ and $x = 2$. Also sketch the graph of the function and interpret it.

(b) Define L'Hopital rule for $x \to 0$ and $x \to \infty$. Find the limit by applying the hypothesis of L'Hopital rule, $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$. **(15)**

(c) If $y = (\sinh^{-1}x)^2$, then show that $(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+n^2y_n=0$ by using Leibnitz's theorem and hence find $(y_n)_0$. **(15)**

2. (a) Let, $f(x) = (x^{2/3} - 1)^3$, (i) find the x-coordinates of all critical points, stationary points and inflection points (ii) find the intervals on which f is increasing and decreasing, (iii) the open intervals on which f is concave up and concave down, (iv) locate all points in a graph to show increasing/decreasing and concavity region. **$(16\frac{2}{3})$**

(b) A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of \$220 per unit. If the total production cost (in dollars) for x units is, **(15)**

$$C(x) = 500{,}000 + 80x + 0.003x^2$$

and if the production capacity of the firm is at most 32,000 units in a specified time, how many units of antibiotic must be manufactured and sold in that time to maximize the profit?

(c) State Taylor's theorem with Lagranges's and Cauchy's from of remainder. Expand $\ln(1 + \sin x)$ at $x = 0$ in power of x with Cauchy's form of remainder. **(15)**

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3. (a) State Rolle's theorem and Mean-value theorem. A function $f(x)$ is defined as follows, **($16\frac{2}{3}$)**

$$f(x)=\begin{cases} x\sin\frac{1}{x} & when |x|<1 \\ 0 & when\, x=0 \end{cases}$$

Show that Mean-value theorem is not applicable for $f(x)$ on (-1, 1).

(b) State Euler's theorem for the homogeneous function $u = f(x, y)$ of degree n and then show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial y\partial x}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u.$ **(15)**

(c) Find tangent, normal, subtangent, subnormal, length of tangent and length of normal to the curve $(x-1)(x-2)y-x+5=0$ at the point of x-interception. **(15)**

4. (a) Define radius and center of curvature for cartesian and polar co-ordinates. Find radius of curvature at (0, 2) of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1.$ **($16\frac{2}{3}$)**

(b) Find all the asymptotes of the curve $x^3+x^2y-x^2-y^3+2xy+2y^2-3x+y=0$. **(15)**

(c) Find the envelop of the family of parabola $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$, where $a^n+b^n=c^n, a, b$ being parameters. **(15)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE** questions.

5. (a) Discuss different types of solution methods of indefinite integral and **($6\frac{2}{3}$+10)**

solve: $\int\frac{\sin 2x}{(a^2\sin^2 x+b^2\cos^2 x)^3}dx.$

(b) Evaluate the integral: $\int\frac{x^2}{\sqrt{x^2-2x+2}}dx.$ **(15)**

(c) Assume two trigonometric functions $f(x)$ and g(x) such that sum of their square is one and $f(0) = 0$. Now, evaluate $\int\frac{dx}{4f(x)+3g(x)+13}$. **(15)**

6. (a) Discuss define integral with some properties. Also, derive the Walli's sine formulas. **($6\frac{2}{3}$+10)**

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(b) Evaluate: $\lim_{n\to\infty}\left[\left(\frac{1}{na}\right)+\left(\frac{1}{na+1}\right)+\left(\frac{1}{na+2}\right)+\ldots\ldots\ldots\ldots+\left(\frac{1}{nb}\right)\right]$. **(15)**

(c) Compute the value of the integral: **(15)**

$$\int_0^{\frac{\pi}{4}} \ln(1+\tan x)dx.$$

7. (a) Define beta and gamma function. Also, prove that **($6\frac{2}{3}$+10)**

$$\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma(m+n)}.$$

(b) Find the value of the improper integral: **(15)**

$$\int_0^1 \frac{\ln(1-x)}{x}dx.$$

(c) Show that

$$\int_0^1 x^5\sqrt{\frac{1+x^2}{1-x^2}}dx=\frac{\pi}{8}+\frac{1}{3}.$$

8. (a) Find the total area and arc length of all the loops of the curve $r = a\cos 2\theta$. **($16\frac{2}{3}$)**

(b) Find the surface area of the solid generated by revolving **(15)**

$$\left(\frac{x}{4}\right)^{\frac{2}{3}}+\left(\frac{y}{3}\right)^{\frac{2}{3}}=1 \text{ about x-axis.}$$

(c) Use cylindrical shells to find the volume of the solid that is generated when the region enclosed by $y = x^3$, $y = 1$, $x = 0$ is revolved about the line $y = 1$. **(15)**

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